# Heat Transfer

Professional Engineer Examination Review – Mechanical

Center for Continuing Engineering Education (C2E2)
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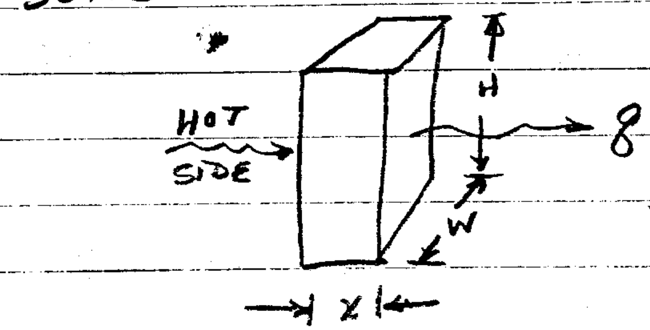
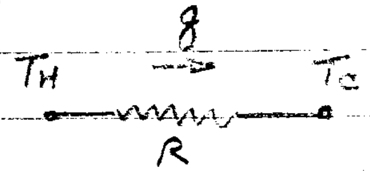
# P.E. HEAT TRANSFER REVIEW

## ELECTRICAL ANALOGY:

SLAB

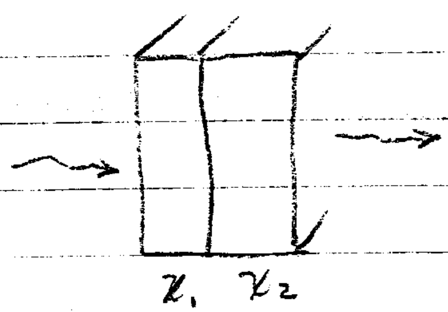



$E = I \times R$

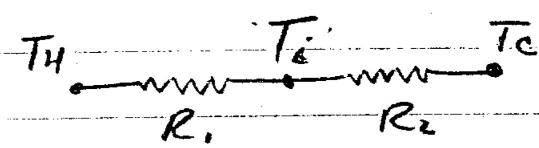
$I = \dfrac{E}{R}$ $\qquad q = \dfrac{T_H - T_C}{R} = \dfrac{KA}{x} \Delta T = UA\,\Delta T$

$$R = \frac{x}{KA}$$




$R_{TOT} = R_1 + R_2$

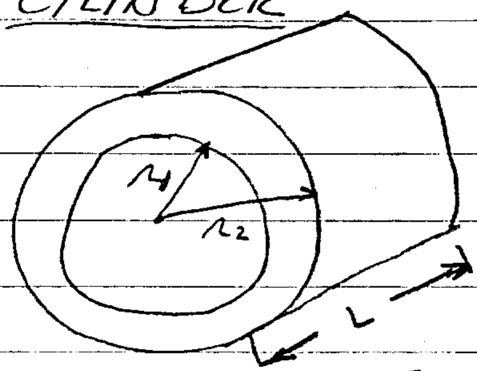
$R_{TOT} = \dfrac{x_1}{K_1 A} + \dfrac{x_2}{K_2 A} = \dfrac{1}{UA}$

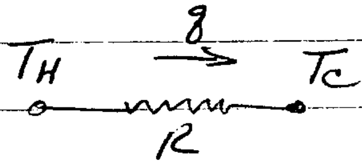



$$q = \frac{T_H - T_i}{R_1} = \frac{T_i - T_C}{R_2} = \frac{T_H - T_C}{R_{TOT}}$$

$$T_i = T_H - qR_1 = T_C + qR_2$$

## CYLINDER




$$R = \frac{\ln \frac{r_2}{r_1}}{2\pi L K}$$

1 DEM. S/S SCAB

## EXAMPLE 1.1 Heat Transfer through Insulation

A refrigerated container is in the form of a cube with 2 m sides and has 5 mm–thick aluminum walls insulated with a 10 cm layer of cork. During steady operation, the temperatures on the inner and outer surfaces of the container are measured to be −5°C and 20°C, respectively. Determine the cooling load on the refrigerator.

### Solution

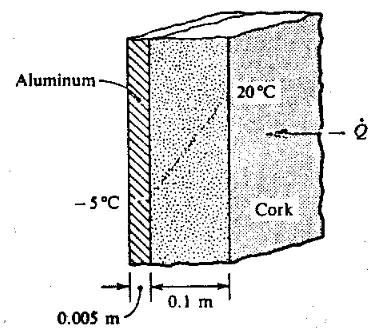
***Given:*** Aluminum container insulated with 10 cm–thick cork.

***Required:*** Rate of heat gain.

***Assumptions:***
1. Steady state
2. One-dimensional heat conduction (ignore corner effects)

Equation (1.10) applies:

$$\dot{Q} = \frac{\Delta T}{R_A + R_B} \qquad \text{where } R = \frac{L}{kA}$$




Let subscripts $A$ and $B$ denote the aluminum wall and cork insulation, respectively. Table 1.1 gives $k_A$ = 204 W/m K, $k_B$ = 0.043 W/m K. We suspect that the thermal resistance of the aluminum wall is negligible, but we will calculate it anyway. For one side of area $A$ = 4 m$^2$, the thermal resistances are

$$R_A = \frac{L_A}{k_A A} = \frac{(0.005 \text{ m})}{(204 \text{ W/m K})(4 \text{ m}^2)} = 6.13 \times 10^{-6} \text{ K/W}$$

$$R_B = \frac{L_B}{k_B A} = \frac{(0.10 \text{ m})}{(0.043 \text{ W/m K})(4 \text{ m}^2)} = 0.581 \text{ K/W}$$

Since $R_A$ is five orders of magnitude less than $R_B$, it can be ignored. The heat flow for a temperature difference of $T_1 - T_2 = 20 - (-5) = 25$ K, is

$$\dot{Q} = \frac{\Delta T}{R_B} = \frac{25 \text{ K}}{0.581 \text{ K/W}} = 43.0 \text{ W}$$

For six sides, the total cooling load on the refrigerator is 6.0 × 43.0 = 258 W.

1 DCM. S/S CYL

## EXAMPLE 2.1 Heat Loss from an Insulated Steam Pipe

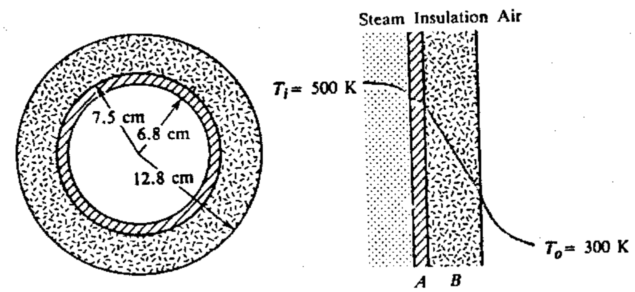
A mild steel steam pipe has an outside diameter of 15 cm and a wall thickness of 0.7 cm. It is insulated with a 5.3 cm–thick layer of 85% magnesia insulation. Superheated steam at 500 K flows through the pipe, and the inside heat transfer coefficient is 35 W/m$^2$ K. Heat is lost by convection and radiation to surroundings at 300 K, and the sum of outside convection and radiation coefficients is estimated to be 8 W/m$^2$ K. Find the rate of heat loss for a 20 m length of pipe.

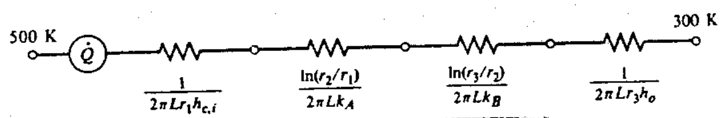
**Solution**

***Given:*** Steam pipe with 85% magnesia insulation.

***Required:*** Heat loss for 20 m length if $h_o$ = 8 W/m$^2$ K.

***Assumptions:*** Steady one-dimensional heat flow.




Equation (2.16) applies, with Eq. (2.17) used to obtain the $UA$ product,

$$\dot{Q} = UA(T_i - T_o)$$

$$R_T = \frac{1}{UA} = \frac{1}{2\pi L}\left(\frac{1}{r_1 h_{c,i}} + \frac{\ln(r_2/r_1)}{k_A} + \frac{\ln(r_3/r_2)}{k_B} + \frac{1}{r_3 h_o}\right)$$

Tables A.1*b* and A.3 in Appendix A give the variation of conductivity with temperature for 1010 steel and magnesia, respectively. As a first step, we guess that the steel is close to the steam temperature (500 K), and since most of the temperature drop will be across the magnesia insulation, its average temperature will be about (500 + 300)/2 = 400 K. The corresponding conductivity values are $k_A$ = 54 W/m K and $k_B$ = 0.073 W/m K.

$$\frac{1}{UA} = \frac{1}{(2)(\pi)(20)}\left(\frac{1}{(0.068)(35)} + \frac{\ln(0.075/0.068)}{54} + \frac{\ln(0.128/0.075)}{0.073} + \frac{1}{(0.128)(8)}\right)$$

$$= \frac{1}{125.7}(0.42 + 0.002 + 7.32 + 0.98)$$

$$UA = 14.4 \text{ W/K}$$

$$\dot{Q} = UA\Delta T = (14.4)(500 - 300) = 2880 \text{ W}$$

Since the resistance of the steel wall is negligible, we do not need to check our guess for its conductivity. For the magnesia insulation, we estimate its average temperature by examining the relevant segment of the thermal circuit. For convenience, the thermal resistance of the insulation is split in half to estimate an average temperature $\overline{T}$:

$$\overline{T} - T_o = \dot{Q}\left[\left(\frac{1}{2}\right)\frac{\ln(r_3/r_2)}{2\pi L k_B} + \frac{1}{2\pi L r_3 h_o}\right]$$

$$\overline{T} - 300 = (2880)\left[\left(\frac{1}{2}\right)\frac{7.32}{125.7} + \frac{0.98}{125.7}\right]$$

$$= 106 \text{ K}$$

$$\overline{T} = 406 \text{ K}$$

A look at Table A.3 shows that our guess of 400 K introduced an error of less than 1%, so there is no need to calculate a new value of $\dot{Q}$ using an improved $k$ value.

LUMPED MODEL

## EXAMPLE 1.6 Quenching of a Steel Plate

A steel plate 1 cm thick is taken from a furnace at 600°C and quenched in a bath of oil at 30°C. If the heat transfer coefficient is estimated to be 400 W/m² K, how long will it take for the plate to cool to 100°C? Take $k$, $\rho$, and $c$ for the steel as 50 W/m K, 7800 kg/m³, and 450 J/kg K, respectively.

### Solution

***Given:*** Steel plate quenched in an oil bath.

***Required:*** Time to cool from 600°C to 100°C.

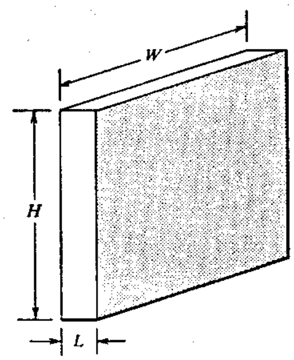
***Assumptions:*** Lumped thermal capacity model valid.

First the Biot number will be checked to see if the lumped thermal capacity approximation is valid. For a plate of width $W$, height $H$, and thickness $L$,

$$\frac{V}{A} \simeq \frac{WHL}{2WH} = \frac{L}{2}$$

$$Bi = \frac{\bar{h} V}{A k_s}$$

where the surface area of the edges has been neglected.




$$\mathrm{Bi} = \frac{\bar{h}_c(L/2)}{k_s}$$

$$= \frac{(400\ \mathrm{W/m^2\,K})(0.005\ \mathrm{m})}{50\ \mathrm{W/m\,K}}$$

$$= 0.04 < 0.1$$

so the lumped thermal capacity model is applicable. The time constant $t_c$ is

$$t_c = \frac{\rho V c}{\bar{h}_c A} = \frac{\rho(L/2)c}{\bar{h}_c} = \frac{(7800\ \mathrm{kg/m^3})(0.005\ \mathrm{m})(450\ \mathrm{J/kg\,K})}{(400\ \mathrm{W/m^2\,K})} = 43.9\ \mathrm{s}$$

Substituting $T_e = 30°\mathrm{C}$, $T_0 = 600°\mathrm{C}$, $T = 100°\mathrm{C}$ in Eq. (1.38) gives

$$\frac{100-30}{600-30} = e^{-t/43.9}$$

Solving,

$$t = 92\ \mathrm{s}$$

### Comments

The use of a constant value of $h_c$ may be inappropriate for heat transfer by natural convection or radiation (see Section 1.5.2).

FINS

## EXAMPLE 2.5 Fins to Cool a Transistor

An array of eight aluminum alloy fins, each 3 mm wide, 0.4 mm thick, and 40 mm long, is used to cool a transistor. When the base is at 340 K and the ambient air is at 300 K, how much power do they dissipate if the combined convection and radiation heat transfer coefficient is estimated to be 8 W/m² K? The alloy has a conductivity of 175 W/m K.

**Solution**

$$\eta_f = \frac{\tanh(\beta L)}{\beta L}$$

***Given:*** Aluminum fins to cool a transistor.

***Required:*** Power dissipated by 8 fins.

***Assumptions:*** 1. Heat transfer coefficient constant along fin.
2. Heat loss from fin tip negligible.

For one fin,

$$A_c = (0.003)(0.0004) = 1.2 \times 10^{-6}\ \text{m}^2$$

$$\mathscr{P} = 2(0.003 + 0.0004) = 6.8 \times 10^{-3}\ \text{m}$$

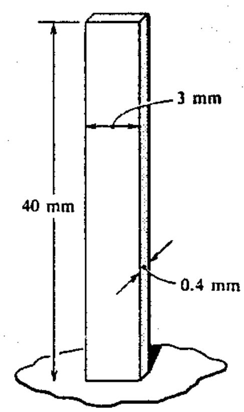
$$\beta^2 = \frac{h\mathscr{P}}{kA_c}$$

$$= \frac{(8.0\ \text{W/m}^2\ \text{K})(6.8 \times 10^{-3}\ \text{m})}{(175\ \text{W/m K})(1.2 \times 10^{-6}\ \text{m}^2)}$$

$$= 259\ \text{m}^{-2}$$

$$\beta = 16.1\ \text{m}^{-1}$$

$$\chi = \beta L = (16.1\ \text{m}^{-1})(0.040\ \text{m}) = 0.644$$




Substituting in Eq. (2.42),

$$\eta_f = \frac{1}{0.644}\tanh(0.644) = \frac{1}{0.644}\frac{e^{2(0.644)} - 1}{e^{2(0.644)} + 1} = 0.881$$

The side surface area of one fin is $\mathscr{P}L = (6.8 \times 10^{-3})(0.040) = 2.72 \times 10^{-4}\ \text{m}^2$. If each fin were 100% efficient, it would dissipate

$$\dot{Q}_{MAX} = h(\mathscr{P}L)(T_B - T_e) = (8)(2.72 \times 10^{-4})(340 - 300) = 8.70 \times 10^{-2}\ \text{W}$$

Since the fins are only 88.1% efficient,

$$\dot{Q} = (0.881)(8.70 \times 10^{-2}) = 7.67 \times 10^{-2}\ \text{W}$$

For 8 fins, $\dot{Q}_{total} = (8)(7.67 \times 10^{-2}) = 0.613$ W.

$$h_R \cong 4\epsilon\sigma T_m^3 \qquad T_m = \frac{(T_1 + T_2)}{2}$$

$$\cong 6 \times \epsilon \ @\ 25°C$$

RADIATION "h" FOR MODERATE TEMPERATURES

CONVECTION:

$$q = h A (T_H - T_C)$$

RADIATION: (NORMAL CASE)

$$q = \sigma \epsilon_1 A (T_H^4 - T_C^4)$$

$$h_R = \frac{\sigma \epsilon_1 (T_H^4 - T_C^4)}{T_H - T_C}$$

$$\cong \sigma \epsilon_1 4 \left[\frac{(T_H + T_C)}{2}\right]^3$$

FORCED – TURBULENT

**EXAMPLE 19.3**

Water enters a 3-cm-diam tube with a velocity of 50 m/s and a temperature of 20°C and is heated. Calculate the average unit convective coefficient. Calculate Re to determine the flow regime.

$$\mathrm{Re} = \frac{(500)(0.03)}{1.006 \times 10^{-6}} = 1.491 \times 10^6 \qquad \therefore \text{turbulent}$$

$$\mathrm{Pr} = 7.0 \qquad \lambda = 0.597$$

Use equation (19.19) to obtain

$$\mathrm{Nu} = 0.023(1.491 \times 10^6)^{0.8}(7.0)^{0.4}$$

$$\mathrm{Nu} = 4350$$

$$\bar{h}_c = \frac{(4350)(0.597)}{(0.03)} = 8.656 \times 10^4 \ \mathrm{W/m^2 \cdot K}$$

## USING "PETUKHOV" FORMULATION

$$f = (.79 \ln Re - 1.64)^{-2}$$

$$= (.79 \ln 1.491 \times 10^{+6} - 1.64)^{-2}$$

$$= .0109$$

$$Nu = \frac{f/8 \, (Re - 1000) \, Pr}{1 + 12.7 \, (f/8)^{.5} \, (Pr^{2/3} - 1)}$$

$$= \frac{.0109/8 \, (1.491 \times 10^{6} - 1000) \cdot 7.0}{1 + 12.7 \left(\frac{.0109}{8}\right)^{.5} (7^{2/3} - 1)}$$

$$= \frac{14211}{2.247} = 6325$$

$$h = 1.26 \times 10^{5} \; W/m^2 K$$

TUBE BANK IN A HEX



When there are fewer than 10 rows of tubes in the streamwise direction, a correction factor must be applied to the result computed from Equation 12.4.13. These correction factors are presented in Table 12.10.

**Table 12.10 Correction factor for Eq.12.4.13 for less than 10 rows of tubes**

| $N$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| Aligned | 0.64 | 0.80 | 0.87 | 0.90 | 0.92 | 0.94 | 0.96 | 0.98 | 0.99 |
| Staggered | 0.68 | 0.75 | 0.83 | 0.89 | 0.92 | 0.95 | 0.97 | 0.98 | 0.99 |

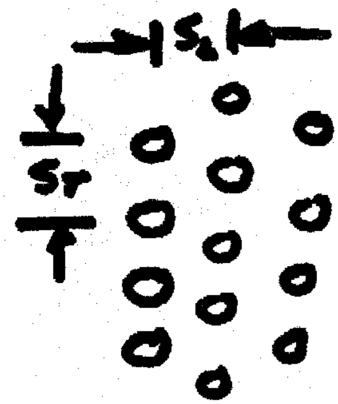
## Example 12.11

Air flows over a bank of tubes in a heat recovery unit. The air enters at 600 F with a velocity of 25 ft/s. The 0.5 in. diameter tubes have an average surface temperature of 300 F and are spaced in a staggered fashion with $S_T = 1.0$ in. and $S_L = 1.5$ in. What is the average heat transfer coefficient on the air side?

**Solution.** From the geometry of the staggered arrangement, $S_D$ is the hypotenuse of the right triangle with sides of length $S_L$ and $S_T/2$. Thus

$$S_D = \sqrt{S_L^2 + S_T^2/4} = \sqrt{2.25 + 0.25} = 1.58''$$

Since $S_D > (S_T + D)/2 = 0.75$ in., the minimum flow section in the bank is at $S_T$, and the maximum velocity is




$$V_{max} = \frac{S_T}{S_T - D} V_\infty = \frac{1.0}{0.5} 25 = 50 \text{ ft/s}$$

The properties of the air evaluated at the film temperature (450 F) are

$$\nu = 1.50 \text{ ft}^2/\text{hr}$$
$$k = 0.0235 \text{ Btu/hr-ft-F}$$
$$\text{Pr} = 0.684$$

and the $\text{Re}_{max}$ is

$$\text{Re}_{max} = \frac{50 \text{ ft/s } (0.5/12) \text{ ft}}{1.50 \text{ ft}^2/\text{hr} \times 1\text{hr}/3600 \text{ s}} = 5000$$

With $S_T/D = 1.0/0.5 = 2.0$ and $S_L/D = 1.5/0.5 = 3.0$, Table 12.9 gives the constants $C = 0.440$ and $m = 0.562$. Thus, Eq.12.4.13 gives the average Nu as

$$\overline{\text{Nu}}_D = 1.13 \times 0.440 \times 5000^{0.562} \times 0.684^{1/3}$$
$$= 52.5$$

Then the heat transfer coefficient for 10 or more rows of tubes is

$$h = \frac{\text{Nu}k}{D} = \frac{52.5 \times 0.0235}{0.5/12} = 30 \text{ Btu/hr-ft}^2\text{-F}$$

$$Re_{max} = \frac{V_{max}D}{\nu}$$

If the fluid approaches the bank with a velocity $V_\infty$, then the maximum velocity generally will occur at the transverse section and

$$V_{max} = \frac{S_T}{S_T - D}V_\infty$$

However, for the staggered arrangement, the maximum velocity may occur at $S_D$ and it will when $S_D < (S_T + D)/2$. In this case

$$V_{max} = \frac{S_T}{2(S_D - D)}V_\infty$$

The correlation for the average Nusselt number is of the form:

$$\overline{Nu}_D = \frac{\bar{h}D}{k} = 1.13C\,Re^m_{max}\,Pr^{1/3} \tag{12.4.13}$$

where the constants C and $m$ depend only on the relative spacing of the tubes. These constants are listed in Table 12.9. All fluid properties in Equation 12.4.13 should be evaluated at the film temperature.

**Table 12.9 Constants $C$ and $m$ for use in Eq.12.4.13 (10 or more rows of tubes)**

| | $S_T/D$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **Aligned** | 1.25 | | 1.5 | | 2.0 | | 3.0 | |
| $S_L/D$ | C | $m$ | C | $m$ | C | $m$ | C | $m$ |
| 1.25 | 0.348 | 0.592 | 0.275 | 0.608 | 0.100 | 0.704 | 0.0633 | 0.752 |
| 1.50 | 0.367 | 0.586 | 0.250 | 0.620 | 0.101 | 0.702 | 0.0678 | 0.744 |
| 2.00 | 0.418 | 0.570 | 0.299 | 0.602 | 0.229 | 0.632 | 0.198 | 0.648 |
| 3.00 | 0.290 | 0.601 | 0.357 | 0.584 | 0.374 | 0.581 | 0.286 | 0.608 |
| **Staggered** | | | | | | | | |
| $S_L/D$ | C | $m$ | C | $m$ | C | $m$ | C | $m$ |
| 0.600 | — | — | — | — | — | — | 0.213 | 0.636 |
| 0.900 | — | — | — | — | 0.446 | 0.571 | 0.401 | 0.581 |
| 1.000 | — | — | 0.497 | 0.558 | — | — | — | — |
| 1.125 | — | — | — | — | 0.478 | 0.565 | 0.518 | 0.560 |
| 1.500 | 0.518 | 0.556 | 0.505 | 0.554 | 0.519 | 0.556 | 0.522 | 0.562 |
| 2.000 | 0.404 | 0.572 | 0.416 | 0.568 | 0.482 | 0.556 | 0.449 | 0.570 |
| 3.000 | 0.310 | 0.592 | 0.356 | 0.580 | 0.440 | 0.562 | 0.428 | 0.574 |

Table 35.3 Natural Convection Film Coefficients: Simplified Equations for Air (isothermal surfaces, 1 atm)

| configuration | GrPr | simplified equation | |
|---|---|---|---|
| vertical plate or vertical cylinder[a] | $10^4$ to $10^9$ | $h = (1.37)\left(\frac{T_s - T_\infty}{L}\right)^{\frac{1}{4}}$ | [SI] |
| | | $h = (0.29)\left(\frac{T_s - T_\infty}{L}\right)^{\frac{1}{4}}$ | [U.S.] |
| | $10^9$ to $10^{12}$ | $h = (1.24)(T_s - T_\infty)^{\frac{1}{3}}$ | [SI] |
| | | $h = (0.19)(T_s - T_\infty)^{\frac{1}{3}}$ | [U.S.] |
| horizontal cylinder | $10^3$ to $10^9$ | $h = (1.32)\left(\frac{T_s - T_\infty}{d}\right)^{\frac{1}{4}}$ | [SI] |
| | | $h = (0.27)\left(\frac{T_s - T_\infty}{d}\right)^{\frac{1}{4}}$ | [U.S.] |
| | $10^9$ to $10^{12}$ | $h = (1.24)(T_s - T_\infty)^{\frac{1}{3}}$ | [SI] |
| | | $h = (0.18)(T_s - T_\infty)^{\frac{1}{3}}$ | [U.S.] |
| horizontal plate: hot surface facing up or cold surface facing down (square)[b] | $10^5$ to $2 \times 10^7$ | $h = (1.32)\left(\frac{T_s - T_\infty}{d}\right)^{\frac{1}{4}}$ | [SI] |
| | | $h = (0.27)\left(\frac{T_s - T_\infty}{L}\right)^{\frac{1}{4}}$ | [U.S.] |
| | $2 \times 10^7$ to $3 \times 10^{10}$ | $h = (1.52)(T_s - T_\infty)^{\frac{1}{3}}$ | [SI] |
| | | $h = (0.22)(T_s - T_\infty)^{\frac{1}{3}}$ | [U.S.] |
| horizontal plate: cold surface facing up or hot surface facing down (square)[b] | $3 \times 10^5$ to $3 \times 10^{10}$ | $h = (0.59)\left(\frac{T_s - T_\infty}{d}\right)^{\frac{1}{4}}$ | [SI] |
| | | $h = (0.12)\left(\frac{T_s - T_\infty}{L}\right)^{\frac{1}{4}}$ | [U.S.] |

[a]A vertical cylinder can be considered a vertical plate as long as $d/L \geq 35/(\mathrm{Gr}_L)^{\frac{1}{4}}$.
[b]For horizontal circular disc, use $L = 0.9 \times$ disc diameter.

cylinders at room temperature (i.e., 70°F (21°C)) and in the range $10^4 < \mathrm{GrPr} < 10^9$.

$$h = (127)\left(\frac{T_s - T_\infty}{L}\right)^{\frac{1}{4}} \quad \text{[SI]} \qquad 35.15(a)$$

$$h = (26)\left(\frac{T_s - T_\infty}{L}\right)^{\frac{1}{4}} \quad \text{[U.S.]} \qquad 35.15(b)$$

*Example 35.1* NAT. CONV.

A horizontal 4.0 in (10 cm) diameter (nominal) pipe carries steam through a 20 ft (6 m) long room. The temperature of the exterior of the pipe surface is 300°F (150°C). The temperature of the air in the room is 100°F (36°C). What is the natural convective heat loss from the exterior of the pipe?

$$h = 1.32\left[\frac{(150-36)}{.1}\right]^{.25} = 7.7$$

*SI Solution*

The characteristic length is the pipe outside diameter.

$$L = 0.10 \text{ m}$$

The temperature gradient is

$$T_s - T_\infty = 150°\text{C} - 36°\text{C} = 114°\text{C}$$

The air properties are evaluated at the film temperature.

$$T_h = \left(\tfrac{1}{2}\right)(T_s - T_\infty) = \left(\tfrac{1}{2}\right)(150^\circ\text{C} + 36^\circ\text{C})$$
$$= 93^\circ\text{C}$$

The air properties at 93°C are found from App. 35.D.

$$\begin{aligned} k &= 0.03115\ \text{W/m·K} \\ \rho &= 0.964\ \text{kg/m}^3 \\ \mu &= 2.15\times 10^{-5}\ \text{kg/s·m} \\ \beta &= 2.74\times 10^{-3}\ \text{1/K} \\ \text{Pr} &= 0.694 \end{aligned}$$

The Grashof number is

$$\text{Gr} = \frac{L^3 g\beta\rho^2(T_s - T_\infty)}{\mu^2}$$
$$= \frac{(0.10\ \text{m})^3\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)\left(2.74\times 10^{-3}\ \frac{1}{\text{K}}\right)\times\left(0.964\ \frac{\text{kg}}{\text{m}^3}\right)^2(114\ \text{K})}{\left(2.15\times 10^{-5}\ \frac{\text{kg}}{\text{s·m}}\right)^2}$$
$$= 6.16\times 10^6$$

$$\text{GrPr} = (6.16\times 10^6)(0.694) = 4.28\times 10^6$$

Using Eq. 35.12 and values from Table 35.2,

$$h = \frac{kC(\text{GrPr})^n}{L}$$
$$= \frac{\left(0.03115\ \frac{\text{W}}{\text{m·K}}\right)(0.53)(4.28\times 10^6)^{\frac{1}{4}}}{0.10\ \text{m}}$$
$$= 7.51\ \text{W/m}^2\text{·K} \qquad h \approx 1.32\left[\frac{150-36}{.1}\right]^{.25}$$

The heat transfer from the pipe is $\approx 7.67$

$$Q = qA = \pi dLh(T_s - T_\infty)$$
$$= \pi(0.10\ \text{m})(6\ \text{m})\left(7.51\ \frac{\text{W}}{\text{m}^2\text{·K}}\right)(150^\circ\text{C} - 36^\circ\text{C})$$
$$= 1614\ \text{W}$$

*Customary U.S. Solution*

The characteristic length is the pipe outside diameter.

$$L = \frac{4\ \text{in}}{12\ \frac{\text{in}}{\text{ft}}} = 0.333\ \text{ft}$$

The temperature gradient is

The air properties are evaluated at the film temperature.

$$T_h = \left(\tfrac{1}{2}\right)(T_s + T_\infty) = \left(\tfrac{1}{2}\right)(300^\circ\text{F} + 100^\circ\text{F})$$
$$= 200^\circ\text{F}$$

The air properties at 200°F are found from App. 35.C.

$$\begin{aligned} k &= 0.0174\ \text{Btu-ft/hr-ft}^2\text{-}^\circ\text{F} \\ \rho &= 0.060\ \text{lbm/ft}^3 \\ \mu &= 1.44\times 10^{-5}\ \text{lbm/ft-sec} \\ \beta &= 1.52\times 10^{-3}\ 1/^\circ\text{F} \\ \text{Pr} &= 0.72 \end{aligned}$$

The Grashof number is

$$\text{Gr} = \frac{L^3 g\beta\rho^2(T_s - T_\infty)}{\mu^2}$$
$$= \frac{(0.333\ \text{ft})^3\left(32.2\ \frac{\text{ft}}{\text{sec}^2}\right)\left(1.52\times 10^{-3}\ \frac{1}{^\circ\text{F}}\right)\times\left(0.060\ \frac{\text{lbm}}{\text{ft}^3}\right)^2(200^\circ\text{F})}{\left(1.44\times 10^{-5}\ \frac{\text{lbm}}{\text{ft-sec}}\right)^2}$$
$$= 6.28\times 10^6$$

Using Eq. 35.12 and values from Table 35.2,

$$h = \frac{kC(\text{GrPr})^n}{L}$$
$$= \frac{\left(0.0174\ \frac{\text{Btu-ft}}{\text{hr-ft}^2\text{-}^\circ\text{F}}\right)(0.53)(4.52\times 10^6)^{\frac{1}{4}}}{0.333\ \text{ft}}$$
$$= 1.28\ \text{Btu/hr-ft}^2\text{-}^\circ\text{F} \qquad h \approx .27\left[\frac{300-100}{.333}\right]^{.25}$$

The heat transfer from the pipe is $\approx 1.34$

$$Q = qA = \pi dLh(T_s - T_\infty)$$
$$= \pi(0.333\ \text{ft})(20\ \text{ft})\left(1.28\ \frac{\text{Btu}}{\text{hr-ft}^2\text{-}^\circ\text{F}}\right)\times(300^\circ\text{F} - 100^\circ\text{F})$$
$$= 5356\ \text{Btu/hr}$$

## 15. FILM COEFFICIENTS FOR AIR ON HEATED FLAT PLATES

If the film coefficient for air heated on a flat plate is known for either the vertical or horizontal configuration, an approximate film coefficient for the corresponding configuration can be determined from Eqs. 35.16 and 35.17.

$$h_{\text{horizontal,facing upward}} \approx 1.27 h_{\text{vertical}} \qquad 35.16$$

35.17

HORIZONTAL CYLINDER – USING APPROX. EQN FOR AIR

FROM ASHRAE FUNDEMENTALS OR REVIEW MANL

U.S. UNITS $L(FT)$, $\Delta T(°F)$ TABLE 10.6 – 10.7

REV. MANL EXPL

$$N_{GR} \times N_{PR} = N_{RA} = 4.5 \times 10^{6} \qquad N_{GR} = 6.28 \times 10^{6}$$

$$h = .27 \times \left(\frac{200}{.333}\right)^{.25} = 1.34 \quad (1.28)$$

EXPL 4.6

$$h = 1.07 \left(\frac{\Delta T}{x}\right)^{.25} \qquad 10^{4} < G_R < 10^{9}$$

$$h = 1.3 \, (\Delta T)^{.333} \qquad 10^{9} < G_R < 10^{12}$$

$$h = 1.07 \times \left(\frac{200}{.3}\right)^{.25} = 5.44 \quad (5.58) \quad W/m^2 K$$

NAT CONV.

**EXAMPLE 4.6** Heat Loss from a Steam Pipe

A 30 cm–O.D. horizontal steam pipe has an outer surface temperature of 500 K and is located in still air at 300 K. Calculate the average heat transfer coefficient and the convective heat loss per meter length of pipe.

**Solution**

***Given:*** Natural convection from a horizontal steam pipe.

***Required:*** $\bar{h}_c$ and heat loss per meter.

Evaluate air properties at the mean film temperature of (500+300)/2 = 400 K. From Table A.7, $k$ = 0.0331 W/m K, $\nu = 25.5 \times 10^{-6}$ m²/s, Pr = 0.69, and $\beta$ = 1/400 for an ideal gas. The Rayleigh number is

$$\mathrm{Ra}_D = \mathrm{Gr}_D\mathrm{Pr} = \frac{\beta \Delta T g D^3}{\nu^2}\mathrm{Pr} = \frac{(1/400)(200)(9.81)(0.3)^3(0.69)}{(25.5 \times 10^{-6})^2} = 1.41 \times 10^8$$

Equation (4.87) applies:

$$\overline{\mathrm{Nu}}_D = 0.36 + \frac{0.518\mathrm{Ra}_D^{1/4}}{[1 + (0.559/\mathrm{Pr})^{9/16}]^{4/9}} = 0.36 + \frac{0.518(1.41 \times 10^8)^{1/4}}{[1 + (0.559/0.69)^{9/16}]^{4/9}} = 42.9$$

$$\bar{h}_c = \left(\frac{k}{D}\right)\overline{\mathrm{Nu}}_D = \left(\frac{0.0331}{0.3}\right)42.9 = 4.73 \text{ W/m}^2\text{ K}$$

$$\dot{Q} = h_c A(T_s - T_e) = (4.73)(\pi)(0.3)(1)(500 - 300) = 892 \text{ W/m}$$

We will check this result using the general correlation for laminar natural flows, Eq. (4.91).

For $L = \pi D/2 = 0.47$ m,

$$\overline{\mathrm{Nu}}_L = 0.52(\mathrm{Gr}_L\mathrm{Pr})^{1/4} = (0.52)\left(\frac{(1/400)(200)(9.81)(0.47)^3(0.69)}{(25.5 \times 10^{-6})^2}\right)^{1/4} = 79.3$$

$$\bar{h}_c = \left(\frac{k}{L}\right)\mathrm{Nu}_L = \left(\frac{0.0331}{0.47}\right)79.3 = 5.58 \text{ W/m}^2\text{ K}$$

**Comments**

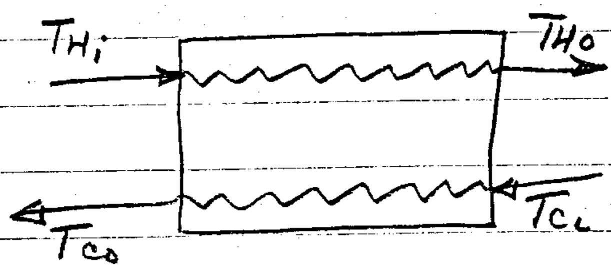
1. The more approximate Eq. (4.91) gives a value of $\bar{h}_c$ that is 18% higher than that from Eq. (4.87).

$$h_r = 4\epsilon\sigma T_m^3$$

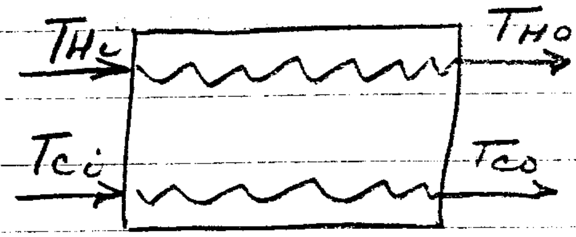
$$= 4 \times .9 \times 5.67 \times 10^{-8} \times 400^3$$

$$= 13.1 \text{ W/m}^2\text{ K}$$

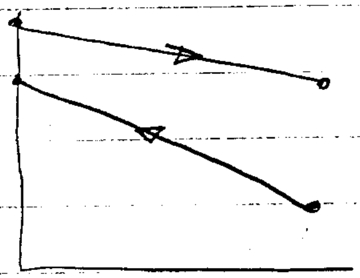
# HEAT EXCHANGER TERMS

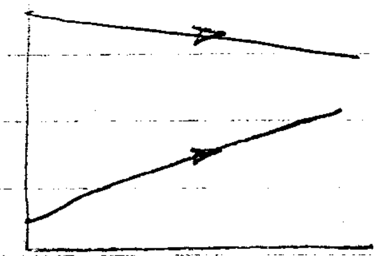



COUNTERFLOW




PARALLEL FLOW





$$\underset{\text{(COUNTER FLOW)}}{LMTD} \equiv \frac{(T_{Hi} - T_{Co}) - (T_{Ho} - T_{Ci})}{\ln \dfrac{T_{Hi} - T_{Co}}{T_{Ho} - T_{Ci}}}$$

$$q = UA\,LMTD = \dot{m}_H c_{p_H}(T_{Hi} - T_{Ho}) = \dot{m}_C c_{p_C}(T_{Co} - T_{Ci})$$

$$C \equiv \dot{m} c_p$$

$$q = C_H(T_{Hi} - T_{Ho}) = C_C(T_{Co} - T_{Ci})$$

$$\epsilon = \text{EFFECTIVENESS} \equiv \frac{q_{ACT}}{q_{MAX}}$$

$$= \frac{C_H \, \Delta T_H}{C_{MIN}(T_{Hi} - T_{Ci})} = \frac{C_C \, \Delta T_C}{C_{MIN}(T_{Hi} - T_{Ci})}$$

$$q_{MAX} = C_{MIN}\,(T_{Hi} - T_{Ci})$$

$$R \equiv \frac{C_{MIN}}{C_{MAX}}$$

$$NTU = \frac{UA}{C_{MIN}}$$

$$\epsilon = f\,(R, NTU, \text{FLOW ARRANGEMENT})$$

FOR $R = 0$ (CONDENSER, EVAPORATOR)

$$\epsilon = 1 - e^{-NTU}$$

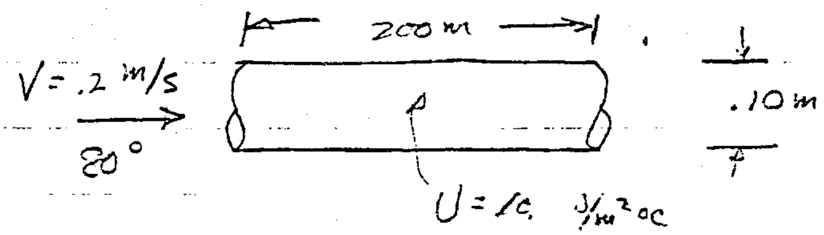
$$NTU = \ln \frac{1}{1-\epsilon}$$

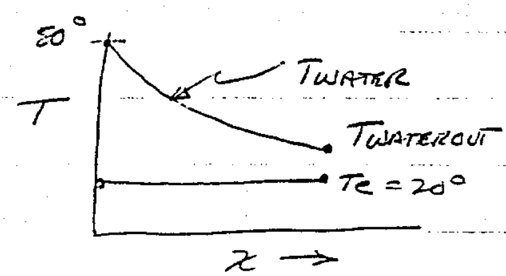
EXPL. PROB. - SINGLE STREAM HEX

A HOT WATER PIPE IS IN SURROUNDINGS WHERE $T_e = 20°C$. WATER ENTERS THE PIPE AT 80° AND A VELOCITY OF .2 m/s. THE PIPE IS THIN-WALLED COPPER, 10 CM IN DIAMETER, AND 200 m LONG.

THE OVERALL CONDUCTANCE, U, FROM THE WATER TO THE SURROUNDINGS IS 10. $W/m^2 °C$.

FIND THE WATER OUTLET TEMPERATURE




$$\dot m = \rho A V = 1000\ kg/m^3 \times \pi \times .05^2 \times .2\ m/s = 1.57\ kg/s$$

$$\dot m C_p = C_{MIN} = 1.57 \times 4180 = 6566\ W/°C$$

$$UA = 10 \times \pi \times .1 \times 200 = 628.3$$

$$NTU = \frac{UA}{C_{MIN}} = \frac{628.3}{6566} = .0957$$

$$\epsilon = 1 - e^{-NTU} = 1 - e^{-.0957} = .0913 = \frac{\Delta T_{WATER}}{T_{W,IN} - T_e}$$

$$\Delta T_{WATER} = .0913\,(80 - 20) = 5.5$$

$$\therefore\ T_{WATER\,OUT} = 80 - 5.5 = 74.5°C$$

## EXAMPLE 1.8 Performance of a Steam Condenser

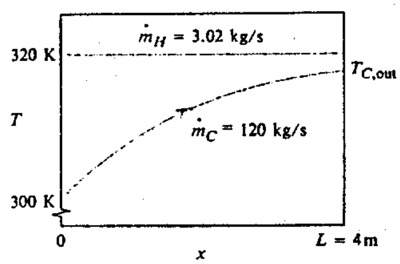
A steam condenser is 4 m long and contains 2000, 5/8 inch nominal-size, 18 gage brass tubes (1.59 cm O.D., 1.25 mm wall thickness). In a test 120 kg/s of coolant water at 300 K is supplied to the condenser, and when the steam pressure in the shell is 10,540 Pa, condensate is produced at a rate of 3.02 kg/s. Determine the effectiveness of the exchanger and the overall heat transfer coefficient. Take the specific heat of the water to be 4174 J/kg K.

### Solution

***Given:*** A shell-and-tube steam condenser.

***Required:*** The effectiveness, $\varepsilon$, and overall heat transfer coefficient, $U$.

***Assumptions:*** $U$ is constant along the exchanger so that Eq. (1.59) applies.




The hot-stream temperature $T_H$ is the saturation temperature corresponding to the given steam pressure of 10,540 Pa; from steam tables (see Table A.12*a* in Appendix A of this text) $T_{\text{sat}} = 320.0$ K. We first find the coolant water outlet temperature from the exchanger energy balance Eq. (1.52):

$$\dot{m}_C c_{pC}(T_{C,\text{out}} - T_{C,\text{in}}) = \dot{m}_H h_{fg}$$

From steam tables, the enthalpy of vaporization at $T_{\text{sat}} = 320$ K is $h_{fg} = 2.389 \times 10^6$ J/kg.

$$(120 \text{ kg/s})(4174 \text{ J/kg K})(T_{C,\text{out}} - 300 \text{ K}) = (3.02 \text{ kg/s})(2.389 \times 10^6 \text{ J/kg})$$

Solving gives $T_{C,\text{out}} = 314.4$ K.

The effectiveness, $\varepsilon$, is then obtained from Eq. (1.58) as

$$\varepsilon = \frac{T_{C,\text{out}} - T_{C,\text{in}}}{T_{\text{sat}} - T_{C,\text{in}}} = \frac{314.4 - 300}{320 - 300} = 0.720$$

$\mathscr{P}L = A_{HT}$

and the number of transfer units, from Eq. (1.59), is

$$N_{tu} = \ln\frac{1}{1-\varepsilon} = \ln\frac{1}{1-0.720} = 1.27 = \frac{U\mathscr{P}L}{\dot{m}_C c_{pC}}$$

Solving for the $U\mathscr{P}L$ product,

$$U\mathscr{P}L = 1.27\dot{m}_C c_{pC} = (1.27)(120 \text{ kg/s})(4174 \text{ J/kg K}) = 6.36 \times 10^5 \text{ W/K}$$

If we choose to base the overall heat transfer coefficient on the outside of the tubes, then, for $N$ tubes, the heat transfer area $\mathscr{P}L$ is

$$A \approx \mathscr{P}L = N\pi DL = (2000)(\pi)(1.59 \times 10^{-2} \text{ m})(4 \text{ m}) = 400 \text{ m}^2$$

Hence, $U = U\mathscr{P}L/\mathscr{P}L = 6.36 \times 10^5/400 = 1590$ W/m$^2$ K

**EXAMPLE 19.5**

It is desired to cool 5000 lbm/hr of oil, $c_p = 0.8$ Btu/lbm-°F from 250°F to 150°F. Water, $c_p = 1.0$ Btu/lbm-°F, is available with a flowrate of 4500 lbm/hr at a temperature of 50°F. The overall coefficient of heat transfer is 15 Btu/hr-ft²-°F. Determine the length of 1-in.-I.D. tubing required for (a) counterflow (b) parallel-flow heat exchange.

$$q = UA\,\overline{\Delta T}$$

(a) Counterflow heat exchange

For tube-in-tube counterflow heat exchangers, $\overline{\Delta T}$ = LMTD. Determine the water outlet temperature by a first-law analysis. From this we can determine $\Delta T_A$ and $\Delta T_B$.

$$q_{H_2O} = q_{oil}$$

$$\dot{m}c_p(\Delta T)_{H_2O} = \dot{m}_o c_{po}(\Delta T)_{oil}$$

$$(\Delta T)_{H_2O} = \frac{(5000)(0.8)(100)}{(4500)(1.0)} = 88.88°\text{F}$$

$$(T_{H_2O})_{out} = 138.88°\text{F}$$

$$\Delta T_B = 150 - 50 = 100$$

$$\Delta T_A = 250 - 138.88 = 111.12$$

$$\text{LMTD} = \frac{\Delta T_A - \Delta T_B}{\ln(\Delta T_A/\Delta T_B)} = \frac{111.12 - 100}{\ln(111.12/100)}$$

$$\text{LMTD} = 105.46$$

$$q = \dot{m}c_p(\Delta T)_{oil} = (5000)(0.8)(100) = 400{,}000\ \text{Btu/hr}$$

$$A = \frac{q}{(U)(\text{LMTD})} = \frac{400{,}000}{(15)(105.46)} = 252.8\ \text{ft}^2$$

$$A = \pi\, dL; \qquad L = \frac{(252.8)(12)}{(\pi)(1)} = 965.8\ \text{ft}$$

(b) Parallel flow heat exchange

$$\Delta T_A = 250 - 50 = 200$$

$$\Delta T_B = 150 - 138.88 = 11.12$$

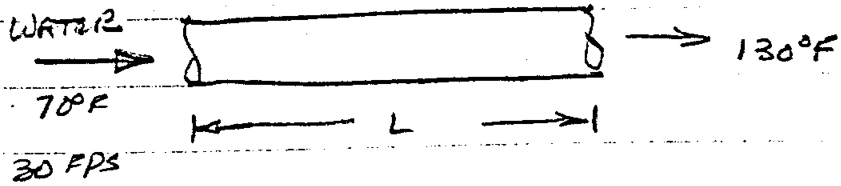
$$\text{LMTD} = \frac{200 - 11.12}{\ln(200/11.12)} = 65.36°\text{F}$$

$$A = \frac{q}{(U)(\text{LMTD})} = \frac{400{,}000}{(15)(65.36)} = 407.9\ \text{ft}^2$$

$$L = \frac{A}{\pi d} = \frac{(407.9)(12)}{(\pi)(1)} = 1558.3\ \text{ft}$$

The parallel-flow configuration requires a 61% area increase to achieve the same heat transfer.

EXPL 10.8 — REVIEW MANUAL




ID = 2"

OD = 2.125"

$T_{WALL}$ = 170°F

USING NTU METHOD — SINGLE STREAM HEX - FIND LENGTH TO REACH 130°

$N_R = 6.76 \times 10^5$

$h = 4144$ BTU/HR-FT² °F (GIVEN)

$\dot{m} = \rho A V$

$= 62 \times \frac{\pi \times 2^2}{4 \times 144} \times 30 = 40.57$ LB/S

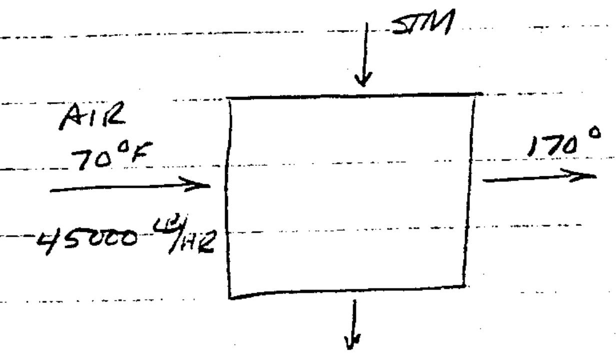
$$NTU = \frac{hA}{\dot{m} c_p} \qquad \epsilon = 1 - e^{-NTU}$$

$$\epsilon = \frac{130-70}{170-70} = .60$$

$$NTU = \ln \frac{1}{1-\epsilon} = .916 = \frac{4144 \times \pi \times \frac{2}{12} \times L}{40.57 \times .998 \times 3600}$$

L = 61.5 FT

"HEAT TRANSFER 2" FROM PEFLEY PG 175




STM - SAT @ 5 PSIG (228°F)

ASSUME $UA \cong (hA)_{AIR}$

PROPOSED TO INCR. AIR 2X WITH SAME TEMP RISE.

WHAT IS NEW $T_{STM}$

$$\epsilon = \frac{170-70}{228-70} = .633 \qquad NTU = \ln \frac{1}{1-\epsilon} = 1.00 = \frac{hA}{\dot{m} c_p}$$

$$\frac{NTU'}{NTU} = \frac{\dfrac{h'A}{(\dot{m} c_p)'}}{\dfrac{hA}{\dot{m} c_p}} = \frac{h'}{h} \frac{\dot{m} c_p}{(\dot{m} c_p)'} = 2^{.8} \times \frac{1}{2} = .871$$

$$NTU' = .871 \qquad \epsilon = 1 - e^{-NTU} = .581 = \frac{100}{T_{SAT} - 70}$$

$$T_{SAT} = 70 + \frac{100}{.581} = 242\,°F \longrightarrow P_{SAT} = 25.9 \text{ PSIA}$$

$$= 11.2 \text{ PSIG}$$

RANGE OF OVERALL "U"

**Table 8.2** Approximate overall heat transfer coefficients.

| Heat Exchanger Duty | $U$, W/m$^2$ K |
|---|---|
| Gas to gas | 10–30 |
| Water to gas (e.g., gas cooler, gas boiler) | 10–50 |
| Condensing vapor-air (e.g., steam radiator, air heater) | 5–50 |
| Steam to heavy fuel oil | 50–180 |
| Water to water | 800–2500 |
| Water to other liquids | 200–1000 |
| Water to lubricating oil | 100–350 |
| Light organics to light organics | 200–450 |
| Heavy organics to heavy organics | 50–200 |
| Air-cooled condensers | 50–200 |
| Water-cooled steam condensers | 1000–4000 |
| Water-cooled ammonia condensers | 800–1400 |
| Water-cooled organic vapor condensers | 300–1000 |
| Steam boilers | 10–40+ radiation |
| Refrigerator evaporators | 300–1000 |
| Steam-water evaporators | 1500–6000 |
| Steam-jacketed agitated vessels | 150–1000 |
| Heating coil in vessel, water to water | |
| Unstirred | 50–250 |
| Stirred | 500–2000 |

**Table 8.1** Recommended values of fouling resistances for heat exchanger design.

| Fluid | Fouling Resistance, $R_f$ $[\mathrm{W/m^2\,K}]^{-1}$ |
|---|---|
| Fuel oil | 0.005 |
| Transformer oil | 0.001 |
| Vegetable oils | 0.003 |
| Light gas oil | 0.002 |
| Heavy gas oil | 0.003 |
| Asphalt | 0.005 |
| Gasoline | 0.001 |
| Kerosene | 0.001 |
| Caustic solutions | 0.002 |
| Refrigerant liquids | 0.001 |
| Hydraulic fluid | 0.001 |
| Molten salts | 0.0005 |
| Engine exhaust gas | 0.01 |
| Steam (non–oil-bearing) | 0.0005 |
| Steam (oil-bearing) | 0.001 |
| Refrigerant vapors (oil-bearing) | 0.002 |
| Compressed air | 0.002 |
| Acid gas | 0.001 |
| Solvent vapors | 0.001 |
| Seawater | 0.0005–0.001 |
| Brackish water | 0.001–0.003 |
| Cooling tower water (treated) | 0.001–0.002 |
| Cooling tower water (untreated) | 0.002–0.005 |
| River water | 0.001–0.004 |
| Distilled or closed-cycle condensate water | 0.0005 |
| Treated boiler feedwater | 0.0005–0.001 |

however the extent of fouling is known only through a measured reduction in heat transfer performance of an exchanger, which is then attributed to a reduced value of the overall heat transfer coefficient. If $U$ is the overall heat transfer coefficient for an unfouled heat exchanger, we can write

$$\frac{1}{U_f \mathscr{P}} = \frac{1}{U \mathscr{P}} + \frac{R_{fH}}{\mathscr{P}_H} + \frac{R_{fC}}{\mathscr{P}_C} \tag{8.9}$$

where $U_f$ is the overall heat transfer coefficient for the fouled exchanger, and $R_{fH}$ and $R_{fC}$ are the hot stream and cold stream **fouling resistance**, respectively. Table 8.1 gives some representative values for fouling resistance. Clearly, the time-dependent nature of the fouling problem is such that it is very difficult to reliably estimate $U$ values if fouling resistances are dominant.

Often one side of a tube wall is finned—for example, the air side of a water-air heat exchanger. The effect of the fins is conveniently included in the overall heat transfer coefficient through use of the fin efficiency. Thus, the overall heat transfer

**EXAMPLE 8.2** Overall Heat Transfer Coefficient for a Condenser

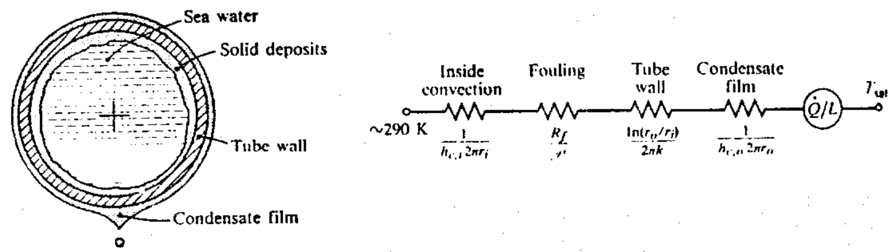
A brass condenser tube has a 30 mm outer diameter and 2 mm wall thickness. Sea water enters the tube at 290 K, and saturated low-pressure steam condenses on the outside of the tube. The inside and outside heat transfer coefficients are estimated to be 4000 and 8000 W/m$^2$ K, respectively, and a fouling resistance of $10^{-4}$ (W/m$^2$ K)$^{-1}$ on the water side is expected. Estimate the overall heat transfer coefficient based on inside area.

### Solution

***Given:*** Brass condenser tube with water-side fouling.

***Required:*** Overall heat transfer coefficient $U$ based on inside area.

***Assumptions:*** The tube wall temperature is ~300 K.




From Table A.1*a*, take $k$ for brass as 111 W/m K. Equation (8.7) gives the $U\mathscr{P}$ product for a clean tube:

$$\frac{1}{U\mathscr{P}} = \frac{1}{h_{c,i}2\pi r_i} + \frac{\ln(r_o/r_i)}{2\pi k} + \frac{1}{h_{c,o}2\pi r_o}$$

$$= \frac{1}{(4000)(2\pi)(0.013)} + \frac{\ln(0.015/0.013)}{2\pi(111)} + \frac{1}{(8000)(2\pi)(0.015)}$$

$$= 10^{-3}(3.06 + 0.21 + 1.33) = 4.60 \times 10^{-3}\ (\text{W/m K})^{-1}$$

The inside perimeter is

$$\mathscr{P} = 2\pi r_i = (2\pi)(0.0130) = 0.0817\ \text{m}$$

Hence,

$$1/U = (0.0817)(4.60 \times 10^{-3}) = 3.76 \times 10^{-4}\ (\text{W/m}^2\,\text{K})^{-1}; \qquad U = 2660\ \text{W/m}^2\,\text{K}$$

Then, from Eq. (8.9) for the fouled tube,

$$\frac{1}{U_f} = \frac{1}{U} + R_{fc} = 10^{-4}(3.76 + 1); \qquad U_f = 2100\ \text{W/m}^2\,\text{K}$$

### Comments

1. The fouling reduces the overall heat transfer coefficient by 21%.
2. Due to fouling, the use of $2\pi r_i$ in the inside convective resistance may be inappropriate.

## EXAMPLE 1.5 Air Temperature Measurement

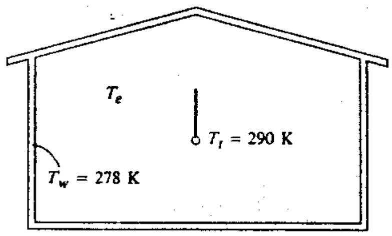
A machine operator in a workshop complains that the air-heating system is not keeping the air at the required minimum temperature of 20°C. To support his claim, he shows that a mercury-in-glass thermometer suspended from a roof truss reads only 17°C. The roof and walls of the workshop are made of corrugated iron and are not insulated; when the thermometer is held against the wall, it reads only 5°C. If the average convective heat transfer coefficient for the suspended thermometer is estimated to be 10 W/m$^2$ K, what is the true air temperature?

### Solution

*Given:* Thermometer reading a temperature of 17°C.

*Required:* True air temperature.

*Assumptions:* Thermometer can be modeled as a small gray body in large, nearly black surroundings at 5°C.




Let $T_t$ be the thermometer reading, $T_e$ the air temperature, and $T_w$ the wall temperature. Equation (1.35) applies,

$$\dot{Q}_{conv} + \dot{Q}_{rad} = 0$$

since at steady state there is no conduction within the thermometer. Substituting from Eqs. (1.24) and (1.18),

$$\bar{h}_c A(T_t - T_e) + \varepsilon\sigma A(T_t^4 - T_w^4) = 0$$

From Table 1.3, $\varepsilon = 0.8$ for pyrex glass. Canceling $A$,

$$10(290 - T_e) + (0.8)(5.67)(2.90^4 - 2.78^4) = 0$$

Solving,

$$T_e = 295 \text{ K} \simeq 22°\text{C}$$

### EXAMPLE 1.2 Heat Loss from a Transistor

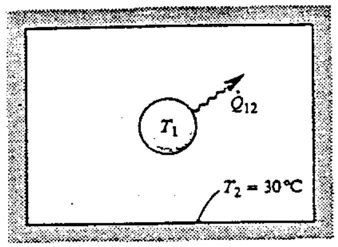
An electronic package for an experiment in outer space contains a transistor capsule, which is approximately spherical in shape with a 2 cm diameter. It is contained in an evacuated case with nearly black walls at 30°C. The only significant path for heat loss from the capsule is radiation to the case walls. If the transistor dissipates 300 mW, what will the capsule temperature be if it is (i) bright aluminum and (ii) black anodized aluminum?

**Solution**

*Given:* 2 cm–diameter transistor capsule dissipating 300 mW.

*Required:* Capsule temperature for (i) bright aluminum and (ii) black anodized aluminum.

*Assumptions:* Model as a small gray body in large, nearly black surroundings.




Equation (1.18) is applicable with

$$\dot{Q}_{12} = 300\,\text{mW}$$

$$T_2 = 30°\text{C} = 303\,\text{K}$$

and $T_1$ is the unknown.

$$\dot{Q}_{12} = \varepsilon_1 A_1(\sigma T_1^4 - \sigma T_2^4)$$

$$0.3\text{ W} = (\varepsilon_1)(\pi)(0.02\text{ m})^2[\sigma T_1^4 - (5.67 \times 10^{-8}\text{ W/m}^2\text{ K}^4)(303\text{ K})^4]$$

Solving,

$$\sigma T_1^4 = 478 + \frac{239}{\varepsilon_1}$$

(i) For bright aluminum ($\varepsilon = 0.035$ from Table 1.3),

$$\sigma T_1^4 = 478 + 6828 = 7306\text{ W/m}^2$$

$$T_1 = 599\text{ K }(326°\text{C})$$

(ii) For black anodized aluminum ($\varepsilon = 0.80$ from Table 1.3),

$$\sigma T_1^4 = 478 + 298 = 776\text{ W/m}^2$$

$$T_1 = 342\text{ K }(69°\text{C})$$

**Comments**

1. The anodized aluminum gives a satisfactory operating temperature, but a bright aluminum capsule could not be used since 326°C is far in excess of allowable operating temperatures for semiconductor devices.
2. Note the use of kelvins for temperature in this radiation heat transfer calculation.